Reg. No. :

Question Paper Code : 21523

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013.

Fourth Semester

Electronics and Communication Engineering

MA 2261/MA 45/MA 1253/10177 PR 401/080380009 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulation 2008/2010)

Time : Three hours | Maximum : 100 marks

Use of Statistical Tables is permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A random variable X has cdf

$$F_X(x) = \begin{cases} 0 & ; \quad x < 1 \\ \tfrac{1}{2}(x-1) & ; \quad 1 \le x < 3 \\ 1 & ; \quad x \ge 3. \end{cases}$$

Find the pdf of X and the expected value of X.

2. Find the moment generating function of binomial distribution.

3. The joint pmf of two random variables X and Y is given by

$$p_{X,Y}(x, y) = \begin{cases} kxy, & x = 1, 2, 3 ; y = 1, 2, 3 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the value of the constant k.

4. The joint pdf of a random variable (X,Y) is $f_{xy}(x,y) = xy^2 + \frac{x^2}{8}, 0 \le x \le 2$, $0 \le y \le 1$. Find $P\{X < Y\}$.

5. Define wide sense stationary process.

6. Show that a binomial process is Markov.

7. A random process $X(t)$ is defined by $X(t) = K \cos wt, t \geq 0$ where w is a constant and K is uniformly distributed over (0, 2). Find the auto correlation function of $X(t)$.

8. Define cross correlation function of $X(t)$ and $Y(t)$. When do you say that they are independent?

9. Define a linear time invariant system.

10. State the convolution form of the output of a linear time invariant system.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A random variable X has pdf

$$f_X(x) = \begin{cases} kx^2 e^{-x} & ; \ x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find the r^{th} moment of X about origin. Hence find the mean and variance. (8)

(ii) A random variable X is uniformly distributed over (0, 10). Find

(1) $P(X < 3), P(X > 7)$ and $P(2 < X < 5)$

(2) $P(X = 7)$. (8)

Or

(b) (i) An office has four phone lines. Each is busy about 10% of the time. Assume that the phone lines act independently.

(1) What is the probability that all four phones are busy?

(2) What is the probability that atleast two of them are busy? (6)

(ii) Describe gamma distribution. Obtain its moment generating function. Hence compute its mean and variance. (10)

12. (a) (i) Two independent random variables X and Y are defined by

$$f_X(x) = \begin{cases} 4ax & ; \ 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad f_Y(y) = \begin{cases} 4\,by & ; \ 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Show that $U = X + Y$ and $V = X - Y$ are uncorrelated. (8)

(ii) State and prove the central limit theorem for in the case of iid random variables. (8)

Or

(b) (i) The equations of two regression lines are $3x + 12y = 19$ and $3y + 9x = 46$. Find $\bar{x}$, $\bar{y}$ and the Correlation Coefficient between X and Y. (8)

(ii) Given the joint pdf of X and Y

$$f_{X,Y}(x, y) = \begin{cases} CX(x-y) & ; 0 < x < 2, -x < y < x \\ 0 & \text{otherwise.} \end{cases}$$

(1) Evaluate C.

(2) Find marginal pdf of X.

(3) Find the conditional density of $Y|X$. (8)

13. (a) (i) Define a semi random telegraph signal process and prove that it is evolutionary. (10)

(ii) Mention any three properties each of auto correlation and of cross correlation functions of a wide sense stationary process. (6)

Or

(b) (i) A random process $X(t)$ defined by $X(t) = A\cos t + B\sin t\,; -\infty < t < \infty$ where A and B are independent random variables each of which has a value -2 with probability $\frac{1}{3}$ and a value 1 with probability $\frac{2}{3}$. Show that $X(t)$ is a wide sense stationary process. (8)

(ii) Define a Poisson process. Show that the sum of two Poisson processes is a Poisson process. (8)

14. (a) (i) Define spectral density of a stationary random process $X(t)$. Prove that for a real random process $X(t)$ the power spectral density is an even function. (8)

(ii) Two random processes $X(t)$ and $Y(t)$ are defined as follows :

$X(t) = A\cos(wt + \theta)$ and $Y(t) = B\sin(wt + \theta)$ where A, B and w are constants ; θ is a uniform random variable over $(0, 2\pi)$. Find the cross correlation function of $X(t)$ and $Y(t)$. (8)

Or

(b) (i) State and prove Wiener – Khintchine theorem. (8)

(ii) If the cross power spectral density of $X(t)$ and $Y(t)$ is

$$S_{XY}(w) = \begin{cases} a + \frac{ibw}{\alpha} & ; -\alpha < w < \alpha, \ \alpha > 0 \\ 0 & \text{otherwise} \end{cases}$$ where a and b are constants. Find the cross correlation function. (8)

15. (a) (i) A random process $X(t)$ is the input to a linear system whose impulse function is $h(t) = 2e^{-t} ; t \geq 0$. The auto correlation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$. Find the power spectral density of the output process $Y(t)$. (8)

(ii) A wide sense stationary noise process $N(t)$ has an auto correlation function $R_{NN}(\tau) = Pe^{-3|\tau|}$ where P is a constant. Find its power spectrum. (8)

Or

(b) (i) If the input to a time invariant stable, linear system is a wide sense stationary process, prove that the output will also be a wide sense stationary process. (8)

(ii) Let $X(t)$ be a Wide sense stationary process which is the input to a linear time invariant system with unit impulse $h(t)$ and output $Y(t)$, then prove that

$S_{YY}(w) = |H(w)|^2 S_{XX}(w)$ where $H(w)$ is Fourier transform of $h(t)$.

(8)
